New SAT Online Resources Fundamental Math Review

In this section, we will review the basic arithmetic that you will need to know for the Math Test. When you are confident with these basic math concepts, the SAT Math material becomes a lot easier. We encourage you to use these basic concepts as reference with the New SAT Guide 2.0. Much of the material covered in that book will rely on your understanding of the following principles. The concepts we will review in this section are:

- Properties of Integers
- Factors and Multiples
- Operations
- Fractions
- Ratios, Percentages, Proportions, and Rates
- Exponents and Radicals
- Scientific Notation
- Basic Geometry
 - o Lines
 - o Polygons, Triangles, and Quadrilaterals
 - o Circles

Integers Part 1

Integers are positive and negative whole numbers, such as -2, -1, 0, 1, 2, etc. Fractions and decimals are not integers. **Zero** is an integer, but it is neither positive nor negative.

Integer Properties				
Word	Definition	Examples		
Positive	Greater than zero	2, 7, 23, 400		
Negative	Less than zero	-2, -7, -23, -400		
Even	Divisible by two	4, 18, 2002, 0		
Odd	Not evenly divisible by two	3, 7, 15, 2001		
Prime	Only divisible by itself and 1	2, 3, 5, 7, 11, 19, 23		
Composite	Divisible by numbers other than itself and 1	4, 12, 15, 20, 21		
Consecutive	Follow each other in numerical order	2, 3, 4, 5, 6		

Here is a chart that summarizes types of integers:

Factors and Multiples

A **factor** of a number is a positive integer that divides that number evenly. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12. When you divide 12 by any other integer, the result is not a whole number. For example, $12 \div 5 = 2.4$. Since 2.4 is not a whole number, 5 is not a factor of 12.

Multiples of a number are the product of that number and any positive integer. For example, some multiples of 3 are 6 and 21 because $3 \times 2 = 6$ and $3 \times 7 = 21$.

Factoring or **factorization** is the process of writing a number as a product of its **prime factors**—the factors that are prime numbers. To factor a number, first divide it by any prime factor. Keep dividing the remainder by prime numbers until the remainder is a prime. Keep track of each factor, even if you divide by the same factor twice.

For example, to factor 12, you can first divide by the prime factor 2. $12 \div 2 = 6$. Divide 6 by another prime factor, such as 2, to get 3. Since the remainder, 3, is a prime number, the prime factors of 12 are 2, 2, and 3.



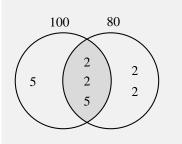
You can start the process with any prime factor, and the factoring will turn out the same. In this example, while there are three prime factors, only two of them are distinct. In the SAT Math, if questions ask for how *many* prime factors a number has, they are looking for the distinct number. For this reason, in this example the correct answer to how many prime factors there are in 12 would be two.

You can find all the factors of a number by multiplying the prime factors by each other. For example, if you multiply the two factors of 2 together, you get 4, which is a factor of 12. Similarly, $2 \times 3 = 6$, so 6 is also a factor of 12.

Types of Factors and Multiples				
Word	Word Definition Ex			
Common factors	Factors that two or more numbers share	6 and 15 have the common factors 1 and 3		
Common multiples	Multiples that two or more numbers share	Some common multiples of 9 and 12 are 36, 108, and 216		
Greatest common factor (GCF)	The largest common factor of two or more numbers	3 is the greatest common factor of 6 and 15		
Least common multiple (LCM)	The smallest common multiple of two or more numbers	36 is the least common multiple of 9 and 12		

Here is a chart that summarizes some types of factors and multiples:

One way to find the GCF and the LCM of two numbers is to organize their prime factors in a Venn diagram like the one below. First, factor each number. Then, write the shared prime factors in the middle of the Venn diagram. Write the remaining prime factors of each number in the correct side of the diagram. The circle under each number contains all the prime factors of that number.



- The factorization of 100 is $2 \times 2 \times 5 \times 5$.
- The factorization of 80 is $2 \times 2 \times 2 \times 2 \times 5$.
- The shared prime factors are 2, 2, and 5.
- The GCF is the product of shared factors: $2 \times 2 \times 5 = 20$.
- The LCM is the product of all the factors in the diagram. Only count the shared factors once: $5 \times 2 \times 2 \times 5 \times 2 \times 2 = 400$.

Operations Part 2

The four main operations in arithmetic are addition, subtraction, multiplication, and division.

Operation	Name of Result	Words	Numbers
Addition	Sum	The sum of 3 and 4 is 7.	3 + 4 = 7
Subtraction	Difference	The difference between 5 and 2 is 3.	5 - 2 = 3
Multiplication	Product	The product of 6 and 4 is 24.	6 × 4 = 24
Division	Quotient	The quotient of 40 divided by 5 is 8.	40 ÷ 5 = 8

If you perform operations with odd or even numbers, you can predict whether the result will be odd or even:

• even + even = even

• $even \times even = even$

• odd + odd = even

• $odd \times odd = odd$

- even + odd = odd
- $even \times odd = even$

Here are some properties you should know for addition and multiplication:

- Associative Property: (a + b) + c = a + (b + c) and a(bc) = (ab)c
- Commutative Property: a + b = b + a and ab = ba
- Distributive Property: a(b + c) = ab + ac

And here are some properties you should know for subtraction and division:

- Subtracting a number is the same as adding its opposite: a b = a + (-b)
- Adding a number to its opposite will give you zero: a + (-a) = 0
- Dividing by a number is the same as multiplying by its reciprocal: $a \div b = a \times \frac{1}{b}$
- Multiplying a number by its reciprocal will give you 1: $a \times \frac{1}{a} = 1$
- You can't divide any number by zero: $a \div 0$ is undefined

Order of Operations

The order of operations is the order that you must follow when carrying out multiple operations:

- **P**arentheses: Do any operations in parentheses first, following the order of operations within the parentheses.
- Exponents: Next, evaluate all exponents.
- Multiplication and Division: Multiply and divide from left to right.
- Addition and Subtraction: Add and subtract from left to right.

You can remember the order of operations with the mnemonic "Please Excuse My Dear Aunt Sally" or the acronym PEMDAS.

Let's see how we'd use the order of operations to solve this calculation:

Example

$$18 \div 3^2 \times 2 - 5 =$$

There are no parentheses in this expression, so move on to the next step and address exponents:

$$18 \div \mathbf{3^2} \times 2 - 5 = 18 \div \mathbf{9} \times 2 - 5$$

Next, address multiplication and division, working left to right:

$$18 \div 9 \times 2 - 5 = 2 \times 2 - 5 = 4 - 5$$

Lastly, address addition and subtraction:

4 - 5 = -1

You have found your answer.

Fractions Part 3

A fraction can be thought of in two ways. First, it is another way to represent division. When we write $\frac{3}{4}$, we mean three divided by four. But more importantly, fractions are used to express parts of a whole.

A fraction often takes the form $\frac{a}{b}$. The number on top, a, is the **numerator**, and the number on the bottom, b, is the **denominator**. Here are some properties of numerators and denominators:

- If a < b, then $\frac{a}{b} < 1$.
- This is called a **proper fraction**. This is called an **improper fraction**. • If a > b, then $\frac{a}{b} > 1$.
- If a = b, then $\frac{a}{b} = 1$.

You can compare two fractions by comparing their numerators and denominators:

- If two fractions have the same *denominator*, the fraction with the larger numerator is the larger fraction: $\frac{6}{7} > \frac{3}{7}$
- If two fractions have the same numerator, the fraction with the smaller denominator is the • larger fraction: $\frac{5}{3} > \frac{5}{7}$

Mixed numbers are improper fractions written as an integer and a fraction. A mixed number is a sum of the integer and the fraction, not the product:

$$2\frac{1}{4} = 2 + \frac{1}{4}$$
 $2\frac{1}{4} \neq 2 \times \frac{1}{4}$

A rational number is a number that can be expressed as a fraction of integers. Some examples include $-\frac{2}{3}$, $5\frac{1}{4}$, and 7. All of these can be expressed as a fraction of integers: $5\frac{1}{4}$ can be written as $\frac{21}{4}$, and 7 can be written as $\frac{7}{1}$.

Irrational numbers cannot be represented as a fraction of integers and will be expressed as a decimal or a symbol. Some examples include π , 81.52602934..., and any other non-repeating, infinite decimal.

Equivalent Fractions

A fraction can be written different ways and still represent the same number. These different fractions with the same value are called **equivalent fractions.** For example, $\frac{1}{2}$ has the same value as $\frac{2}{4}$ and $\frac{10}{20}$. To rewrite a fraction without changing its value, multiply or divide the numerator and the denominator by the same number:

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4} \qquad \qquad \frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

Multiplying the numerator and denominator of a fraction by the same number does not change the value of the fraction because you are really multiplying the fraction by 1:

$$\frac{1 \times 2}{2 \times 2} = \frac{1}{2} \times \frac{2}{2} = \frac{1}{2} \times 1 = \frac{1}{2}$$

A fraction can be **simplified** by dividing its numerator and denominator by their greatest common factor. For example, let's simplify $\frac{60}{105}$. First, we'd factor 60 and 105:

$$60 = 2 \times 2 \times 3 \times 5$$
$$105 = 3 \times 5 \times 7$$

60 and 105 have the common factors 3 and 5, so their greatest common factor is $3 \times 5 = 15$. Therefore, we can simplify the fraction by dividing the numerator and denominator by 15:

$$\frac{60 \div 15}{105 \div 15} = \frac{4}{7}$$

Operations with Fractions

You can only add or subtract two fractions when they have the same denominator—in other words, when they have a **common denominator**.

To add or subtract fractions with the same denominator, add or subtract the numerators and keep the denominator:

$$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$$

If the fractions have different denominators, you must convert them to equivalent fractions with the same denominator. Try using the least common multiple of the denominators.

Let's add $\frac{5}{6} + \frac{3}{8}$. Since the fractions have different denominators, the first step is to convert them to a common denominator. We'll use 24 because it is the least common multiple of 6 and 8:

$$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$
$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

Now that we have two fractions with a common denominator, we can add them:

$$\frac{20}{24} + \frac{9}{24} = \frac{29}{24}$$

To convert mixed numbers to improper fractions, convert the integer to a fraction, then add it to the fraction:

$$2\frac{1}{4} = 2 + \frac{1}{4} = \frac{2 \times 4}{1 \times 4} + \frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$$

You can multiply fractions without converting them to common denominators. Just multiply their numerators and denominators:

$$\frac{5\times7}{3\times8} = \frac{35}{24}$$

To divide by a fraction, multiply by its reciprocal. The **reciprocal** of a fraction is the fraction created when you flip the original numerator and denominator. For example, the reciprocal of $\frac{7}{10}$ is $\frac{10}{7}$. The reciprocal of 3 is the reciprocal of $\frac{3}{1}$, which is $\frac{1}{3}$.

To divide $\frac{7}{10}$ by 3, you need to multiply by the reciprocal of 3:

$$\frac{7}{10} \div 3 = \frac{7}{10} \times \frac{1}{3} = \frac{7}{30}$$

Complex Fractions

Complex fractions are fractions that have one or more fractions in their numerator and/or denominator.

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Example	
	$\frac{2-\frac{1}{2}}{3+\frac{5}{6}}$

To simplify a complex fraction, you can simplify the numerator and denominator and then divide. For example, to simplify the complex fraction above, you can first simplify the numerator:

$$2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

Then, you can simplify the denominator:

$$3 + \frac{5}{6} = \frac{18}{6} + \frac{5}{6} = \frac{23}{6}$$

And finally, you can divide the two fractions:

$$\frac{3}{2} \div \frac{23}{6} = \frac{3}{2} \times \frac{6}{23} = \frac{18}{46} = \frac{9}{23}$$

Fractions and Decimals

If you divide the numerator by the denominator, you can convert a fraction into an integer or a **decimal**, which is a way of representing a fraction out of 10.

$$\frac{3}{4} = 3 \div 4 = 0.75$$

Decimals can be easier to compare than fractions if the fractions have different denominators or different numerators. On the calculator section, you can use a calculator to convert fractions to decimals. For the no-calculator section, you should know the common fraction-decimal conversions:

1 _	1 =	2 _	1	3
$\frac{1}{2} = .5$	$\frac{1}{3} = .3$	$\frac{1}{3} = .6$	$\frac{1}{4} = .25$	$\frac{1}{4} = .75$

The horizontal bar over the decimal means that it repeats infinitely: $.\overline{3} = .33333...$

Ratios, Percentages, Proportions, and Rates

Part 4

Ratios

A **ratio** shows a relationship between two quantities. Ratios can be expressed as a fraction $\left(\frac{1}{4}\right)$, with a colon (1:4), or with the word "to" (1 to 4). Ratios can be converted like fractions by multiplying or dividing each quantity by the same number. For example, if a jar has 12 red marbles and 15 blue marbles, the ratio of red marbles to blue marbles would be $\frac{12}{15}$ or 12:15. This can be reduced to 4:5.

Notice that ratios are not a "part-to-whole" relationship like a fraction unless one quantity is the total. In our jar of marbles, there is a total of 27 marbles. Therefore, if we wanted to write the fraction of marbles that are red, we'd write $\frac{12}{27}$, not $\frac{12}{15}$.

Ratios can compare more than 2 quantities. For example, if a jar has 12 red marbles, 15 blue marbles, and 6 green marbles, the ratio of red to blue to green marbles is 12:15:6 or 4:5:2.

Percentages

A percentage is a ratio that compares a quantity to 100. For example, 20 is 80% of 25 because 20 80 $\frac{1}{25} = \frac{1}{100}$

To convert a percentage to a fraction, re-write it as a fraction of 100 and simplify.

$$45\% = \frac{45}{100} = \frac{9}{20}$$

To convert a percentage to a decimal, divide the percentage by 100:

45% = .45

Proportions

A **proportion** is an equation stating that two fractions are equal. For example, the proportion $\frac{5}{15} = \frac{7}{21}$ shows that $\frac{5}{15}$ and $\frac{7}{21}$ are the same number. They are both multiples of $\frac{1}{3}$.

Proportions will often involve one or more variables. To solve these equations, you can **cross-multiply** by multiplying the numerator of each side by the denominator of the other: if $\frac{a}{b} = \frac{c}{d}$, then ad = cb.

Example

If $\frac{4}{13} = \frac{x}{78}$, what is the value of x?

First, cross-multiply:

$$4(78) = 13x$$

Then, solve for *x*:

$$x = \frac{4(78)}{13} = 24$$

Rates

A **rate** is a fraction that shows the relationship between two quantities with different units. A rate is different than a ratio, which compares quantities within a certain category, like red marbles to blue marbles or boys to girls. Rates often use the word "per," as in cents per pound or miles per hour. The word "in" is also frequently used to predict future rates, as in the temperature will raise 30 degrees in one hour, or that a person will run 5 miles in 35 minutes.

Example

If Jane can read 10 pages in 25 minutes, how many minutes will it take her to read 35 pages?

First, let's say that x is the number of minutes it will take Jane to read 35 pages. We can then set up a proportion:

$$\frac{10 \text{ pages}}{25 \text{ minutes}} = \frac{35 \text{ pages}}{x \text{ minutes}}$$

Then, we can cross-multiply and solve for *x*:

$$10x = 25(35)$$
$$x = \frac{25(35)}{10} = 87.5$$

The correct answer is 87.5 minutes.

When solving problems involving rates, pay attention to the units in the question. The answer may be in different units than are given in the problem. To convert units, set up and solve a proportion between the different units.

Example

If Juan is driving at 50 miles per hour, how many miles does he travel in 12 minutes?

First, set up a proportion to convert the rate to miles per minute:

50 miles		1 hour		5 miles
1 hour	X	60 minutes	=	6 minutes

Then, find the distance travelled in 12 minutes:

$$\frac{5 \text{ miles}}{6 \text{ minutes}} = \frac{x \text{ miles}}{12 \text{ minutes}}$$
$$x = \frac{5(12)}{6} = 10 \text{ miles}$$

Exponents and Radicals Part 5

An exponent indicates that a number is being multiplied by itself a certain number of times. The number being multiplied is called the **base**. The raised digit is the **exponent**, and it tells you how many times a number is being multiplied by itself.

Example

In the expression 3^5 , 3 is the base and 5 is the exponent. This tells you that 3 is being multiplied by itself 5 times:

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3$$

An exponent is also called a **power**. In the example above, we can say that 3^5 is the 5^{th} power of 3.

In the equation $3^2 = 9$, 3 is the **square root** of 9, which we can also write as $\sqrt{9}$. This means that $3 \times 3 = 9$. The symbol we use for square roots ($\sqrt{7}$) is called a **radical**. The number under the square root is called the **radicand**.

 $\sqrt{9}$ is also the equivalent of $9^{\frac{1}{2}}$. The *x*th **root** of any number *a* is written as $a^{\frac{1}{x}}$ or $\sqrt[x]{a}$. For example, $\sqrt[4]{81} = 81^{\frac{1}{4}} = 3$ because $3^4 = 81$.

Exponent Rules

Here is a chart that shows some important rules for exponents and radicals. In this chart, a and b represent any number, and m and n represent any positive integers.

		Exponent Rules
	Rule	Example
$a^1 = a$		$8^1 = 8$
$a^0 = 1$		$8^0 = 1$
$a^{-m} = \frac{1}{a^m}$		$8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

Rule	Example
$a^{\frac{1}{m}} = \sqrt[m]{a}$	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
$a^m a^n = a^{m+n}$	$8^2 \times 8^3 = 8^{2+3} = 8^5 = 32768$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{8^3}{8^2} = 8^{3-2} = 8^1 = 8$
$(a^m)^n = a^{mn}$	$(8^2)^3 = 8^{2 \times 3} = 8^6 = 262144$
$a^m b^m = (ab)^m$	$8^3 \times 2^3 = (8 \times 2)^3 = 16^3 = 4096$
$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$	$\frac{8^3}{2^3} = \left(\frac{8}{2}\right)^3 = 4^3 = 64$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$
$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$	$\sqrt{16 \times 4} = \sqrt{16} \times \sqrt{4} = 4 \times 2 = 8$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$

In the expression $\sqrt[m]{a}$, if *m* is even, *a* must be positive to give a real result. Otherwise the result is expressed as a complex number, a concept that is explained in Section 7 of this chapter.

Remember to use the correct order of operations when you are working with expressions with exponents and roots. First, do all operations inside parentheses. Then simplify all exponents and roots before multiplying, dividing, adding, or subtracting.

Scientific Notation

Place Values

Each digit in a decimal has a **place value**, which refers to where the digit is located in the number. In the number 123, 1 is in the hundreds place, 2 is in the tens place, and 3 is in the ones or units place. Digits to the right of the decimal point also have place values. In the number 0.45, 4 is in the tenths place and 5 is in the hundredths place.

Each place value represents a power of 10. Places to the left of the decimal point are products of 10 to a positive integer, while places to the right of the decimal point are products of 10 to a negative integer.

 $123.45 = 1 \times 10^{2} + 2 \times 10^{1} + 3 \times 10^{0} + 4 \times 10^{-1} + 5 \times 10^{-2}$

You can also write 123.45 as the product of a single decimal and a power of 10:

$$123.45 = 1.2345 \times 10^2$$

Scientific Notation

Place values and powers of 10 are useful for writing very large or very small numbers in a shorter form called scientific notation. **Scientific notation** displays a number as a product of a decimal and a power of 10.

To convert a number to scientific notation, re-write the number as the product of the non-zero digits and the power of ten of the digit occupying the largest position. For example, the largest non-zero digit in 7,100,000 is 7, which is in the millionths place (10⁶). 7,100,000 \div 10⁶ = 7.1, so 7,100,000 = 7.1 × 10⁶.

You can write very small numbers as a product of ten to a negative exponent. For example, the largest non-zero digit in 0.000003409 is the 3 in the millionths place (10^{-6}) . 0.000003409 \div $10^{-6} = 3.409$, so 0.000003409 = 3.409×10^{-6} .

Notice that a number in scientific notation always consists of a decimal whose largest digit is in the ones place multiplied by a power of ten. Large numbers greater than 1 or negative numbers less than -1 are positive powers of 10. Numbers between -1 and 1 are negative powers of 10.

Operations with Scientific Notation

If two or more numbers have the same power of ten, you can add or subtract them by adding or subtracting the decimals:

$$5.4 \times 10^8 - 2.93 \times 10^8 = (5.4 - 2.93) \times 10^8 = 2.47 \times 10^8$$

If the result is larger than or equal to 10 or smaller than 1, you must adjust the power of ten so that the decimal has one digit to the left of the decimal point.

Example

What is the sum of 7.2×10^{-3} and 5.5×10^{-3} ?

First, you add the decimals:

$$7.2 \times 10^{-3} + 5.5 \times 10^{-3} = 12.7 \times 10^{-3}$$

Then, you convert this result to correct scientific notation. Divide the right side by 10, moving the decimal one to the left, and multiplying the 10^{-3} by 10:

$$12.7 \times 10^{-3} = 1.27 \times 10^{-2}$$

If the numbers are multiplied by different powers of 10, you must convert them to standard notation to add or subtract them. Then perform the arithmetic and convert them back to scientific notation.

You can multiply or divide numbers in scientific notation that have different numbers of ten. To multiply numbers in scientific notation, multiply the decimals and add the exponents on the powers of 10. To divide numbers in scientific notation, divide the decimals and subtract the exponents on the powers of 10. Make sure the result is in correct scientific notation.

Example

What is $(2.5 \times 10^{13}) \times (6.8 \times 10^{-5})$?

First, multiply the decimals:

$$2.5 \times 6.8 = 17$$

Then, add the exponents:

$$10^{13} \times 10^{-5} = 10^{13 + (-5)} = 10^{8}$$

The product is therefore $17 \times 10^8 = 1.7 \times 10^9$.

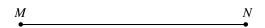
Fundamental Geometry Part 7

Lines

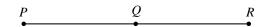
A **line** is a straight, one-dimensional object: it has infinite length but no width. Using any two points, you can draw exactly one line that stretches in both directions forever. For instance, between the points *A* and B below, you can draw the line \overleftrightarrow{AB} . You name a line by drawing a horizontal bar with two arrows over the letters for two points on the line.



A **line segment** is a portion of a line with a finite length. The two ends of a line segment are called **endpoints**. To name a line segment, identify two points on the line, and draw a horizontal bar with above the letters for those two points. For instance, in the figure below, the points M and N are the endpoints of the line segment \overline{MN} .



The point that divides a line segment into two equal pieces is called its **midpoint**. In the figure below, the point *Q* is the midpoint of the line segment \overline{PR} .



Because Q is the midpoint, it divides the segment into two equal pieces. Therefore, you know that PQ = QR. $\overline{PQ} = \overline{QR}$.

Polygons

A **polygon** is a two-dimensional shape with straight sides. Polygons are named for the number of their sides:

Types of Polygons		
mber of Sides		
3		
4		
5		
6		
7		
8		

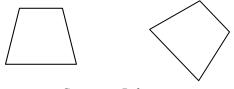
A **vertex** of a polygon is a point where two sides meet. An **interior angle** of a polygon is an angle on the inside of the polygon formed by the intersection of two sides. A **regular polygon** has sides that are all the same length and interior angles that are all the same measure. Make sure to take a look at the New SAT Guide 2.0, Chapter 5, Part 1 for more details on angles and their properties.

To calculate the interior angles of any polygon with *n* sides, use the following formula:

Sum of interior angles = $180^{\circ}(n-2)$

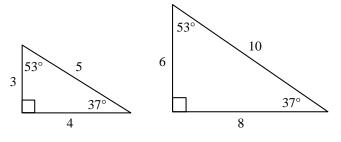
Using this formula, the sum of the interior angles in a hexagon is $180^{\circ}(6-2) = 720^{\circ}$.

Two polygons are **congruent** if they have the same size and shape. Congruent polygons have an equal number of sides, equal lengths of corresponding sides, and equal measures of corresponding interior angles. Congruent polygons have an equal number of sides, equal lengths of corresponding sides, and equal measures of corresponding interior angles. For example, the quadrilaterals below are congruent because they are identical in shape and in size. One just happens to be rotated.



Congruent Polygons

Two polygons are **similar** if they have the same shape, but not the same size. Similar polygons have an equal number of sides, equal measures of corresponding interior angles, and proportional lengths of corresponding sides. For example, the two triangles below are similar because their angles are the same and their sides maintain the same ratio of 3:4:5. However, the triangle on the right is twice as large as the triangle on the left.



Similar Polygons

The **perimeter** of the polygon is the distance around the polygon. To find any polygon's perimeter, add up the lengths of its sides. For example, the triangles above have perimeters of 3 + 4 + 5 = 12 and 6 + 8 + 10 = 24.

The **area** of any polygon is the total space inside a polygon's perimeter. Area is always expressed in terms of square units, such as square inches (in²) or square centimeters (cm²). Because polygons can have different numbers of sides, each type of polygon has its own formula for calculating area. Continue reading to learn about the areas of triangles and quadrilaterals, as well as other special properties of these polygons.

Triangles

A triangle is a polygon with exactly three sides. The interior angles of a triangle add to 180°.

Scalene Isoceles Right

Triangles can be categorized according to their sides and angles:



- In an equilateral triangle, all three sides are the same length and each angle is 60°.
- In an **isosceles** triangle, two of the sides are the same length and the two angles opposite them are congruent.
- In a scalene triangle, all three sides are different lengths and all three angles are different measures.
- In a **right** triangle, two sides of the triangle are perpendicular, creating a right angle.

To find the area of a triangle, multiply its base by its height, which is a line segment perpendicular to the base. Then, divide by two:

Area of a triangle =
$$\frac{\text{base} \times \text{height}}{2}$$

The triangle to the right has a base of 8 units and a height of 6 units, so it has an area of 24 square units:



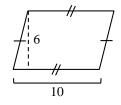
In the New Guide 2.0, we talk about some more special properties of right triangles.

Quadrilaterals

A **quadrilateral** is a polygon with exactly four sides. The interior angles of a quadrilateral add to 360°. Here are some types of quadrilaterals.

A **parallelogram** is a quadrilateral with two sets of parallel sides. The opposite sides of a parallelogram have equal lengths. The area of a parallelogram is equal to its base multiplied by its height, which is a line segment drawn perpendicular to its base. For example, the parallelogram to the right has an area of 60 square units.

Area = base \times height = $6 \times 10 = 60$

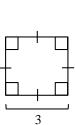


A **rectangle** is a parallelogram with four right angles. Like all parallelograms, the opposite sides of a rectangle are parallel and have equal lengths. The area of a rectangle is equal to its length multiplied by its width. For example, the rectangle to the right has an area of 35 square units:

Area = length
$$\times$$
 width = 5 \times 7 = 35

A **square** is a rectangle with four equal sides. A square is a regular quadrilateral because all sides are the same length and all angles are the same measure (90°). The area of a square is equal to the length of one of its sides squared. For example, the square to the right has an area of 9 square units:

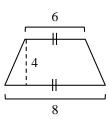
Area = side² =
$$3^2$$
 = 9



7

5

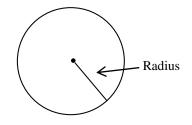
A **trapezoid** is a quadrilateral with only one set of parallel sides. These parallel sides are called the trapezoid's bases. The area of a trapezoid is equal to the sum of its bases divided by two, multiplied by its height. For example, the trapezoid to the right has an area of 28 square units:



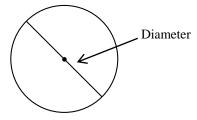
Area =
$$\frac{(\text{base } 1 + \text{base } 2) \times \text{height}}{2} = \frac{(6+8) \times 4}{2} = 28$$

Circles

A **circle** is a two-dimensional figure made up of points that are all the same distance from its center. The line segment drawn from the center of the circle to any point on the circle is called a **radius** (plural: radii). All possible radii of a circle are the same length.



The **diameter** of a circle is a line segment that connects two points on the circle and passes through the center. The length of the diameter of a circle is equal to twice the length of its radius. All diameters of a circle are the same length.



diameter = $2 \times radius$

The **circumference** of a circle is the distance around the circle. It can be found by multiplying the diameter by π (**pi**), a special number equal to approximately 3.14:

circumference = diameter
$$\times \pi$$

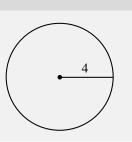
Because π is a non-repeating, non-ending decimal number (3.1415927...), you often leave the symbol π as it is when calculating the circumference or area of a circle. Answers on the SAT will be left in terms of π or will give a number for π (usually 3.14) to use for calculations. This gives a more accurate answer than rounding a lengthy decimal number.

To find the **area** of a circle, multiply π by the circle's radius squared:

area =
$$\pi \times radius^2$$

Example

What is the circumference and area of the circle on the right?



To calculate the circumference, use the radius of 4 to find the diameter and then multiply by π :

circumference = diameter $\times \pi = (2 \times 4) \times \pi = 8\pi$

To calculate the area, square the radius and multiply by π :

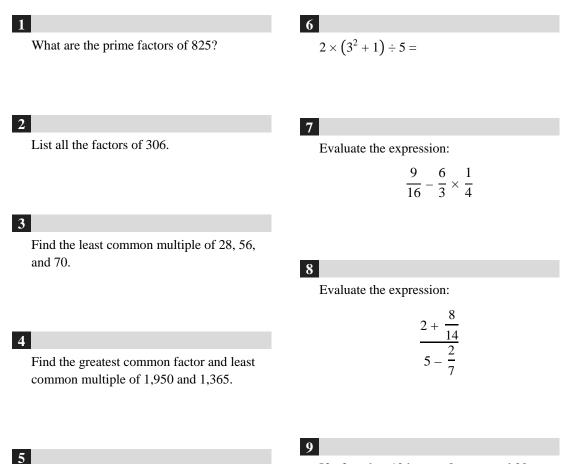
area =
$$\pi \times \text{radius}^2 = \pi \times 4^2 = 16\pi$$

The circumference of the circle is 8π units and its area is 16π units squared.

Fundamental Math Online Drills

Part 8

Now that you've reviewed some basic concepts, take a moment to answer the following drill questions. The following questions are included as a means of solidifying your key concepts, are intended as drills only, and are not representative of what you will find on an exam. Questions on the SAT exam will incorporate these ideas, but in the form of more complicated problems.



If a farm has 12 horses, 8 cows, and 20 chickens, what fraction of these animals are cows?

 $6(3^2 - 8 \times 2) + 1 =$

10 If $\frac{4}{5} = \frac{24}{x}$, what is the value of x?

17

Simplify the following expression:

 $\sqrt{720x^5}$

11

12

3 days?

Sally got 85% of the questions on a test correct. If she got 3 questions wrong, how many questions were on the test?

If a store sells packages of 6 bagels and produces 126 bagels per day, how many

packages of bagels does the store produce in

18

Rewrite the following expression without a radical sign:

 $\sqrt[3]{225y^7}$

19

$$(9.4 \times 10^{-15}) \times (3.0 \times 10^{31}) =$$

13

Brian can walk 5 laps in 8 minutes. At this pace, how many full laps can he walk in 23 minutes?

=

20

If $1.6 \times 10^5 + x = 4.46 \times 10^6$, what is the value of *x*? Write your response in scientific notation.

14
$$(7-3)^{\frac{1}{2}} - 6 \times 2$$

15

16

 $2^5 \times 4^2 =$

 $6^2 \times 6^{-3} \div 6^{\frac{1}{5}} =$

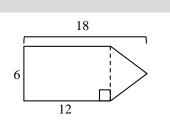
21 *Q R S*

In the figure above, *R* is the midpoint of \overline{QS} . If $\overline{QR} = 6$, what is the length of \overline{QS} ?

22

If the area of a square is 81 m², what is the length of one of the square's sides?

23



24

What is the area of the polygon shown above?

What is the area of the circle above?

25

If the area of a circle is 64π , what is its diameter?

Answers

1.	3, 5, 5, 11	1410
2.	1, 2, 3, 6, 9, 17, 18, 34, 51, 102, 153,	15. 2 ⁹ or 512
	306	16. $6^{-\frac{6}{5}}$
3.	280	17. $12x^2\sqrt{5x}$
4.	GCF: 195 LCM: 13,650	
5.	-41	18. $15^{\frac{2}{3}}y^{\frac{7}{3}}$
6.	4	19. 2.82×10^{17}
7	$\frac{1}{16}$	20. 4.3×10^6
		21. 12
8.	$\frac{6}{11}$	22. 9
9.		23. 90
	5	24. 49π
	30	25. 16
11.	20	
12.	63	
13.	14	